Control Examples

Notes – examples 1&2 on pages 55-58

Example 1: How to use Routh-Hurwitz

- The characteristic equation of a system is:
- $2s^3 + 4s^2 + 4s + 12 = 0$
- Is the system stable or unstable? If it is unstable, how many roots lie in the right half of the s-plane?
- 1st Criterion: Are all of the coefficients a_0, a_1, … ,a_n non-zero?
- Do they have the same sign?
- The answer is yes: on to criterion 2

Example 1

- Creating the Routh Array for: $2s^3 + 4s^2 + 4s + 12 = 0$

s^3	2	4	0
s^2	4	12	0
s			
s^0			

Example 1

- Creating the Routh Array for: $2s^3 + 4s^2 + 4s + 12 = 0$

s^3	2	4	0
s^2	4	12	0
s	$= \frac{4 \times 4 - 2 \times 12}{4}$		
s^0			

Example 1

- Creating the Routh Array for: $2s^3 + 4s^2 + 4s + 12 = 0$

s^3	2	4	0
s^2	4	12	0
s	$= \frac{16 - 24}{4} = -2$	$= \frac{4 \times 0 - 2 \times 0}{4} = 0$	0
s^0			

Example 1

- Creating the Routh Array for: $2s^3 + 4s^2 + 4s + 12 = 0$

s^3	2	4	0
s^2	4	12	0
s	-2	0	0
s^0	12	0	0

- Two sign changes in the left column: system is unstable with two positive roots

Example 2: Adventures in stability

- A 'Three term controller' has the following Transfer Function:

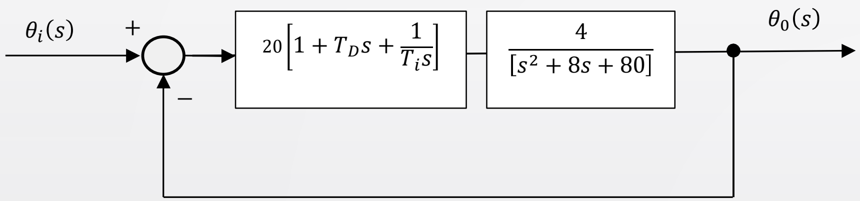
$$\frac{\theta_c(s)}{\theta_\varepsilon(s)} = 20\left[1 + T_D s + \frac{1}{T_i s}\right]$$

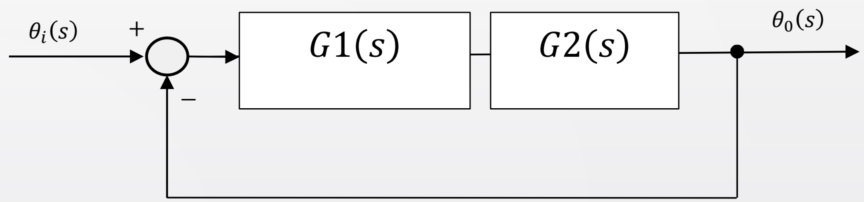
- Where T_D = Derivative time and T_i = Integral time or Reset Time.
- This controller is used to control a process with Transfer Function:

$$\frac{\theta_0(s)}{\theta_c(s)} = \frac{4}{[s^2 + 8s + 80]}$$

- Unity feedback is used.
- If integral action is NOT employed, find the value of T_D required to give a closed loop damping ratio of unity.

Block Diagram

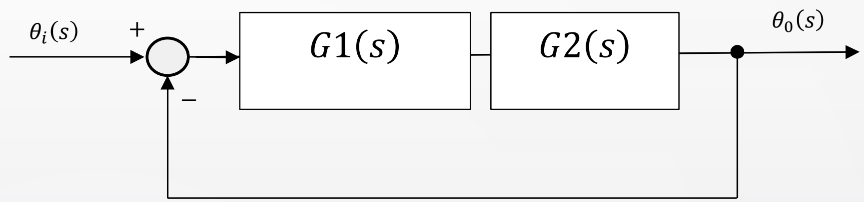




- If you really enjoy algebra …
- $\theta_0(s) = (\theta_i - \theta_0)20\left[1 + T_D s + \frac{1}{T_i s}\right]\frac{4}{(s^2+8s+80)}$
- …
- $\frac{\theta_0(s)}{\theta_i(s)} = \frac{\left[1+T_D s+\frac{1}{T_i s}\right]\frac{80}{(s^2+8s+80)}}{1+\left[1+T_D s+\frac{1}{T_i s}\right]\frac{80}{(s^2+8s+80)}} = \frac{80\left[1+T_D s+\frac{1}{T_i s}\right]}{(s^2+8s+80)+80\left[1+T_D s+\frac{1}{T_i s}\right]}$

- $G1(s) = 20\left[T_D s + 1 + \frac{1}{T_I s}\right]$
- $G2(s) = \frac{4}{[s^2+8s+80]}$
- $\theta_0(s) = (\theta_i - \theta_0)G1(s)G2(s)$
- $\theta_0(s)\big(1 + G1(s)G2(s)\big) = \theta_i(s)G1(s)G2(s)$
- $\frac{\theta_0(s)}{\theta_i(s)} = \frac{G1(s)G2(s)}{1+G1(s)G2(s)} = \frac{\left[1+T_D s+\frac{1}{T_i s}\right]\frac{80}{(s^2+8s+80)}}{1+\left[1+T_D s+\frac{1}{T_i s}\right]\frac{80}{(s^2+8s+80)}} = \frac{80\left[1+T_D s+\frac{1}{T_i s}\right]}{(s^2+8s+80)+80\left[1+T_D s+\frac{1}{T_i s}\right]}$

Block Diagram



- $G1(s) = 20\left[T_D s + 1 + \frac{1}{T_I s}\right]$
- $G2(s) = \frac{4}{[s^2+8s+80]}$
- $\theta_0(s) = (\theta_i - \theta_0)G1(s)G2(s)$
- $\theta_0(s)\big(1 + G1(s)G2(s)\big) = \theta_i(s)G1(s)G2(s)$
- $\frac{\theta_0(s)}{\theta_i(s)} = \frac{G1(s)G2(s)}{1+G1(s)G2(s)} = \frac{\left[1+T_D s+\frac{1}{T_i s}\right]\frac{80}{(s^2+8s+80)}}{1+\left[1+T_D s+\frac{1}{T_i s}\right]\frac{80}{(s^2+8s+80)}} = \frac{80\left[1+T_D s+\frac{1}{T_i s}\right]}{(s^2+8s+80)+80\left[1+T_D s+\frac{1}{T_i s}\right]}$

- If integral action is NOT employed, find the value of T_D required to give a closed loop damping ratio of unity.

$$\frac{\theta_0(s)}{\theta_i(s)} = \frac{80[1 + T_D s]}{(s^2 + 8s + 80) + 80[1 + T_D s]}$$

- Only the denominator is of concern here:
- $s^2 + (8 + 80T_D)s + 160 = s^2 + 2\gamma\omega s + \omega^2$
- $\omega^2 = 160 \qquad \omega = 4\sqrt{10}$
- $\gamma = 1 \qquad 2\omega s = 8\sqrt{10} = 8 + 80T_D$
- $T_D = \frac{8\sqrt{10}-8}{80} = \frac{\sqrt{10}-1}{10} = 0.216s$

- If this value of T_D is used, determine the minimum value of T_i that can be used if the system is to remain stable.

$$\frac{\theta_0(s)}{\theta_i(s)} = \frac{80\left[1 + 0.216s + \frac{1}{T_i s}\right]}{(s^2 + 8s + 80) + 80\left[1 + 0.216s + \frac{1}{T_i s}\right]}$$

- Denominator (characteristic equation) is given by:
- $s^2 + (8 + 80 \times 0.216)s + 160 + \frac{80}{T_i s} = 0$
- $s^3 + 25.28s^2 + 160s + {}^{80}/_{T_i} = 0$
- Time to use Routh-Hurwitz

- $s^3 + 25.28s^2 + 160s + {}^{80}/_{T_i} = 0$
- Time to use Routh-Hurwitz

s^3	1	160	0
s^2	25.28	${}^{80}/_{T_i}$	0
s	b_1	0	0
s^0	${}^{80}/_{T_i}$	0	0

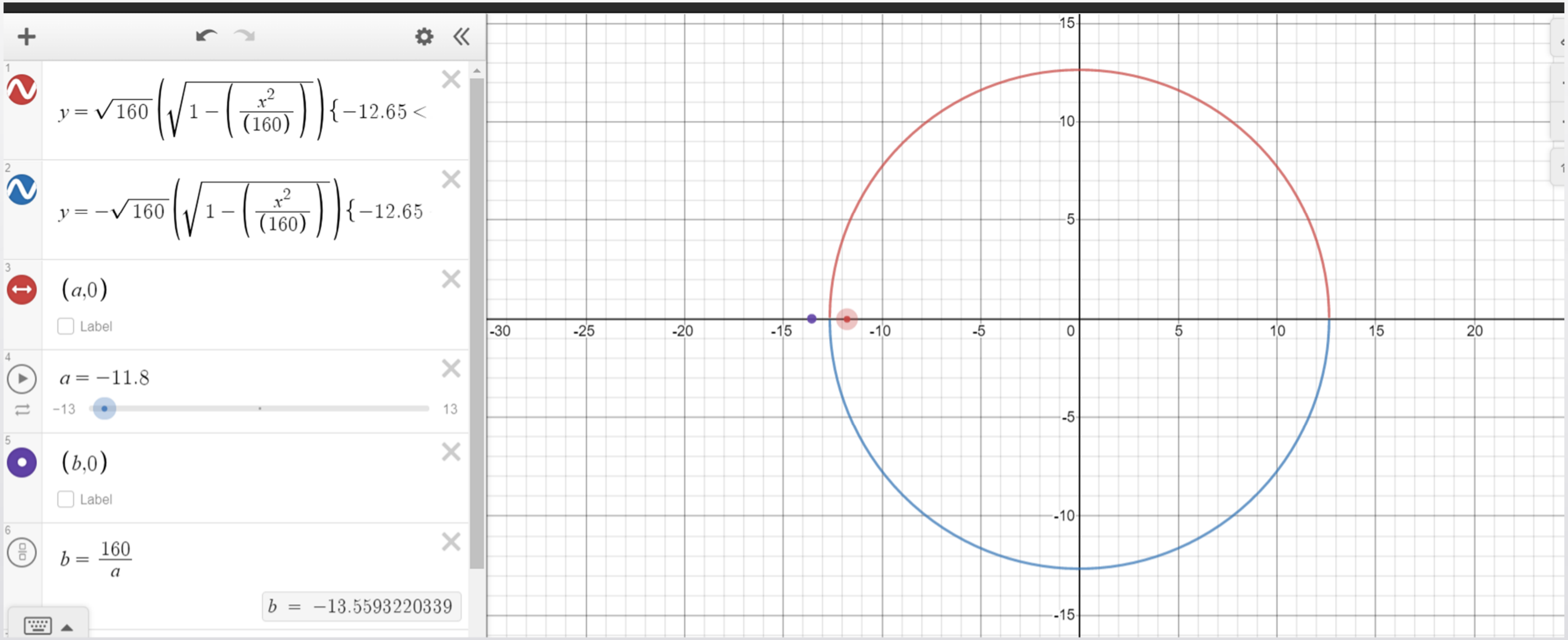
$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} = \frac{25.28 \times 160 - \frac{80}{T_i}}{25.28} = 160 - \frac{3.165}{T_i}$$

s^3	1	160	0
s^2	25.28	$^{80}/_{T_i}$	0
s	$160 - \dfrac{3.165}{T_i}$	0	0
s^0	$^{80}/_{T_i}$	0	0

- For Stability: No change of sign in column 1.
- $80T_i = 0$ $\quad$ $T_i > 0$
- $160 - 3.165/T_i > 0$ i.e. $T_i > 0.01978$
- If $T_i < 0$ then there is one root in the right half of the s-plane
- If $0 < T_i < 0.01978$ then there are two roots in the right half of the s-plane

- When the values of T_D and T_i calculated above are employed, determine the nature of the transient response of the system to an arbitrary input.
- Characteristic equation is now:
- $s^3 + 25.28s^2 + 160s + \frac{80}{0.01978} = s^3 + 25.28s^2 + 160s + 4044.49 = 0$
- At the margin of stability, this will have two imaginary roots and one real root:
- $s^3 + 25.28s^2 + 160s + 4044.49 = (s + \alpha)(s^2 + \omega^2) = s^3 + \alpha\ s^2 + \omega^2 s + \alpha\omega^2$
- α=25.28
- $\omega = \sqrt{160} = 4\sqrt{10} = 12.65$

Visualisation of root locus



System response to a step input: